A - Regular - Minimising the allocation

Note that there is often more than one way to carry out the algorithm and end up with the best allocation/assignment.

For more help visit www.hungarianalgorithm.com

This is the original cost matrix:

66	64	2	12
40	61	8	21
81	74	37	99
94	73	66	53

1. We subtract the row minimum from each row:

64	62	0	10	(-2)
32	53	0	13	(-8)
44	37	0	62	(-37)
41	20	13	0	(-53)

2. We subtract the column minimum from each column:

32	42	0	10
0	33	0	13
12	17	0	62
9	0	13	0
(-32)	(-20)		

3A. Cover all zeros with the **minimum** number of lines:

32	42	0	10
0	33	0	13
12	17	0	62
9	0	13	0

3B. We can cover all zeroes with 3 lines - fewer than 4, so create additional zeros. The smallest uncovered number above is 10. We subtract this number from all uncovered elements and add it to all elements that are covered twice:

22	32	0	0
0	33	10	13
2	7	0	52
9	0	23	0

3A. Cover all zeros with the **minimum** number of lines:

22	32	0	0
0	33	10	13
2	7	0	52
9	0	23	0

4. Because 4 lines are now required to cover all zeroes, the zeros cover an optimal assignment:

22	32	0	0
0	33	10	13
2	7	0	52
9	0	23	0

This corresponds to the following optimal assignment in the original cost matrix:

66	64	2	12
40	61	8	21
81	74	37	99
94	73	66	53

The optimal value equals 162.

B - Missing row or column (unbalanced)

This is the original cost matrix:

66	64	20	12
40	61	30	21
81	74	37	45

1. The cost matrix contains more columns than rows, we add dummy rows with zeros to make the matrix square:

66	64	20	12
40	61	30	21
81	74	37	45
0	0	0	0

2. We subtract the row minimum from each row:

54	52	8	0	(-12)
19	40	9	0	(-21)
44	37	0	8	(-37)
0	0	0	0	

NB Because each column contains a zero, subtracting column minima has no effect.

3A. Cover all zeros with the **minimum** number of lines:

54	52	8	0
19	40	9	0
44	37	0	8
0	0	0	0

3B. We can cover all zeroes with 3 lines - fewer than 4, so create additional zeros. The smallest uncovered number above is 8. We subtract this number from all uncovered elements and add it to all elements that are covered twice:

46	44	0	0
11	32	1	0
44	37	0	16
0	0	0	8

3A. Cover all zeros with the **minimum** number of lines:

46	44	0	0
11	32	1	0
44	37	0	16
0	0	0	8

3B. We can still cover all zeroes with 3 lines, so create additional zeros. We subtract smallest uncovered number of 11 from all uncovered elements and add it to all elements that are covered twice:

35	33	0	0
0	21	1	0
33	26	0	16
0	0	11	19

4. Because 4 lines are now required to cover all zeroes, the zeros cover an optimal assignment:

35	33	0	0
0	21	1	0
33	26	0	16
0	0	11	19

This corresponds to the following optimal assignment in the original cost matrix:

66	64	20	12
40	61	30	21
81	74	37	45

The optimal value equals 89.

C - Maximising the allocation

This is the original cost matrix:

66	64	20
40	61	30
81	74	37

1. Subtract all elements from 81 (the largest element of cost matrix):

15	17	61
41	20	51
0	7	44

Note that we are now minimising the difference from 81, which has the effect of maximising the allocation.

2. We subtract the row minimum from each row:

0	2	46	(-15)
21	0	31	(-20)
0	7	44	

3. We subtract the column minimum from each column:

0	2	15
21	0	0
0	7	13
		(-31)

4A. Cover all zeros with the **minimum** number of lines:

0	2	15
21	0	0
0	7	13

4B. We can cover all zeroes with 2 lines - fewer than 3, so create additional zeros. The smallest uncovered number above is 2. We subtract this number from all uncovered elements and add it to all elements that are covered twice:

0	0	13
23	0	0
0	5	11

4A. Cover all zeros with the **minimum** number of lines:

0	0	13
23	0	0
0	5	11

5. Because 3 lines are now required to cover all zeroes, the zeros cover an optimal assignment:

0	0	13
23	0	0
0	5	11

This corresponds to the following optimal assignment in the original cost matrix:

66	64	20
40	61	30
81	74	37

The optimal value equals 175.

NB If an assignment problem is both unbalanced and requires maximising, it's usually best to carry out subtraction step first (example C) and then add balancing row or column (example B).

Three problems to practice

Three taxis, 1, 2 and 3, are available and there are three customers, A, B and C, requiring taxis. The distances between the taxis and the customers are shown in the table below, in kilometres. The company wishes to assign the taxis to customers so that the total distance travelled is a minimum.

Customers

Taxis		A	B	C
	1	27	18	10
	2	16	15	19
	3	20	14	12

(a) Obtain the opportunity cost matrix.
(b) Test this matrix to decide if an optimal assignment can be made.

This is the optimal assignment:

27	18	**10**
16	15	19
20	**14**	12

The optimal value equals 40.

A book supplier has three salespersons to assign to four regions. The salespersons are able to cover the regions in different amounts of time. The amount of time, in days, required by each salesperson to cover each region is shown in the table below. Which salesperson should be assigned to which region in order to minimise total time? Obtain the optimal assignment by using the Hungarian algorithm and calculate the total time.

Regions

Salesperson		A	B	C	D
	1	10	2	8	6
	2	9	3	11	3
	3	3	1	4	2

This is the optimal assignment:

10	**2**	8	6
9	3	11	**3**
3	1	4	2

The optimal value equals 8.

A head of department has four teachers to be assigned to four different courses. All of the teachers have taught the courses in the past and have been evaluated by the students. The rating for each teacher for each course is given in the table below, a perfect score is 100. The head of department wants to know the optimal assignment of teachers to courses that will maximise the overall average evaluation.

	A	B	C	D
1	85	80	80	75
2	75	85	78	81
3	83	81	70	74
4	81	82	83	78

Use the Hungarian algorithm to solve this assignment problem.

This is the optimal assignment:

85	80	80	75
75	85	78	**81**
83	**81**	70	74
81	82	**83**	78

The optimal value equals 330.